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ABSTRACT

An experience in attempting to transform research into practice is described, with questions raised about the relationship between research and practice. The author begins by describing the two research traditions which exerted the most force on the design of the program. How the philosophy and findings of those traditions were interpreted and implemented is discussed as the SummerMath program is described. This was a six-week program for high school students, to encourage young women to gain an understanding of mathematics in order to increase their participation in mathematics and mathematics-related fields, as well as to implement the findings of research to improve learning. Sections focus on research into practice, research on mathematics learning, curriculum, instruction, women and mathematics, and evaluation. Results of the implementation and new research questions are each presented. (MNS)

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SummerMath: Research into Practice

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
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In choosing to design and implement a six-week summer program in mathematics for young high school women, after working with a major research institute, I gained firsthand experience in both the promise and the mischief of the relations between research and practice. There was unexpected loss--solitude, professional status, relative security, and protection of a research institute--in my choice of trying to "make a difference" to someone. And I, in turn, inherited the fragmentation, relevance, self-doubt, immediacy, morality, and promise of the world of practice.

In this paper, I hope to share with you the experience of attempting to transform research into practice and in doing so attempt to raise questions in your minds about what the relationship between research and practice can be. To do so, I will describe the two research traditions which exerted the most force on the design of the program, how we interpreted and implemented the philosophy and findings of those traditions, what we can say about the results of the implementation, and finally, what new research questions have been borne of these endeavors.

Before doing so, I wish to discuss briefly a perspective on the relationship between research and practice which for me makes sense of what we were attempting. By using this perspective, wherein reason and practice become two different types of practice, I can think of educational research as neither needing to thirst for "scientific" legitimacy nor needing to be required to prescribe immediate action.

Paul Feyerabend (1978) wrote of two traditions, reason and practice. In this context, there are perhaps three traditions: theory, empirical research, and practice. For this discussion I am combining the first two in one category, research, and thinking of them as sharing the characteristics Feyerabend attributes to reason. Feyerabend wrote:

> What is called 'reason' and 'practice' are therefore two different types of practice, the difference being that one clearly exhibits some simply and easily producible formal aspects, thus making us forget the complex and hardly understood properties that guarantee that simplicity and producibility, while the other drowns the formal aspects under a great variety of accidental properties. But complex and implicit reason (as in practice) is still reason, and a practice with simple formal features hovering above a pervasive but unnoticed background of linguistic habits (as in reason) is still a practice. (My parentheticals added.) (p.26)

He described the interaction between traditions in the following way: "We arrive at the idea of a guide who is part of the activity and is changed by it." Thus, he concluded, "Again the problem is not the interaction of a practice with something different and external, but the development of one tradition under the impact of others." (p.33)

Thus, research and practice are two traditions, and when they are juxtaposed, both are affected--when working within one, the other can act as a guide--but there is eventually and happily an interaction which changes meanings within both the traditions as they resonate with and reverberate against each other. It is the merging and juxtaposition of research and practice which gives the SummerMath program, a six-week program for young women in high school, its uniqueness and makes its lessons of value to educational researchers. For nine months the program is planned, and the simple, formal models from research provide our way of looking. Then, for six cacophonous weeks, we are submerged in practice as those models are buffeted about. As we slowly emerge from the activity, we find both our views of the research and the practice have changed as we assess how effective the models and predictions from research were in making sense of our experience and as we plan for the future.

After describing the SummerMath program I will describe how current research was used in designing, implementing, and evaluating a six-week summer program in mathematics. To do so, I will describe the SummerMath program briefly; indicate how two areas of research, that on student misconceptions and on women and mathematics, guided the program development; describe what we know about the effects of the program; and then describe specific new questions which need examination as a result of implementation of this research.

Program Description

In the summer of 1981, Mount Holyoke College decided to develop a summer program in mathematics for high school students. The goal of the program was to encourage young women to gain an understanding of mathematics in order to increase their participation in mathematics and mathematics related fields. The program was also intended to implement the findings of current research in cognitive studies to improve the learning of mathematics and in women and mathematics, to encourage continued participation in mathematics courses.

The staff of the program includes a number of educational researchers. As Director, I came to the program from the Institute for Research on Teaching, with an intent to implement and evolve further the findings on the learning of mathematics. Jack Lochhead was primarily responsible for proposing SummerMath to the administration of the College.

Besides Jack, a number of other members of the Cognitive Development Project at the University of Massachusetts participated in curriculum writing and instruction, including Peter Rosnick, John Clement, Pamela Hardiman, and Ronald Narode. Other major contributions to the curriculum were made by Richard Upchurch, Debbie Schifter, and Felicity Callahan. A clinical psychologist, Dr. Patricia Romney, conducted the workshops in math anxiety. In our current planning for a companion program, SummerMath for Teachers, Joan Mundy has shared in frequent discussions with me on the topics discussed in this paper.

Unlike so many summer programs, SummerMath is targeted at students who are academically capable but for whom mathematics is a hurdle. These students are neither remedial nor gifted and talented. Many are likely to avoid mathematics or to be anxious about mathematics. They may not be representative of a math-anxious population, because in choosing to attend most have recognized this tendency as a weakness and are committed to changing it. Of the forty responses received on the evaluation conducted six weeks after the program, 89% indicated that it was their choice to attend. I suspect the percent would have been lower prior to participation.

SummerMath is a six-week summer program which is devoted to the improvement of mathematical understanding. The curriculum focuses on teaching students to construct mathematical ideas for themselves and is in sharp contrast to materials designed to achieve skill performance only. The students in SummerMath 1982 enrolled in three one and one-half hour courses which met five days per week: Fundamental Mathematics Concepts, Computers, and Laboratory Activities. The philosophy behind all curricular materials and the instruction is constructivist in that students are expected to participate actively in all the learning experiences; the focus is on the processes of analytic and mathematical thought. Such an approach requires students to search for patterns, express those patterns precisely in language and in symbols, and generalize those patterns to new contexts.

In Fundamental Mathematics Concepts, topics from arithmetic, algebra, geometry, and trigonometry were studied. The materials were designed to press for conceptual understanding rather than memorization by stressing a few essential concepts: patterns, unit, number, ratio, and function. In the materials, puzzling problems were used to create a developmental need to revisit or explore new concepts, and the emphasis was placed on process rather than on getting an answer.

In the computing section of the SummerMath program, graphics were used to teach programming concepts and to motivate mathematical insights. The students used twenty-one Apple II Plus computers with the language LOGO, which was developed by Seymour Papert (1980) and his colleagues at

M.I.T. Students were encouraged to pursue independent projects as they learned the concepts of computing, such as modes, variables, procedures, and recursion.

In Laboratory Activities, students learned to see mathematics as integral to other fields: geography, astronomy, statistics, biology, and environmental studies. Groups of students began by working on problems designed by the staff and learned to discover problems of their own making. Learning to ask good questions and pursue mathematical insights was essential to this component of SummerMath.

The program's staff was particularly concerned with increasing the participation of women in mathematics. Of special concern were the factors which discourage women from continuing their education in mathematics. One of these was the belief that mathematics is better suited to males as a career option and hence unnecessary for the future aspirations of women (Armstrong, 1979; Fennema, 1977). Another was that ability and not effort accounts for success in mathematics (Wolleat, et al. 1980). These factors were countered by a concerted effort to discuss the attitudes of the young women toward mathematics and to provide career counseling.

Other aspects of the program included both a recreational and an arts and crafts program. In the arts and crafts program, mathematics and art were united as students created quilting squares, pounded out leaf designs on textiles, made cooperative murals, and constructed geodesic domes and kites. Speakers and evening workshops were also available on topics such as astronomy, hydropower, geology, mathematics education, women and mathematics, and underwater acoustics. For students with a history of mathematics anxiety, a clinical psychologist offered counseling on techniques to relieve anxiety.

Research into Practice

There is a moral dilemma for researchers and teacher educators to face as we document repeatedly the misconceptions students hold about mathematics and the differential effects of our curriculum on young women. We must not only describe the situation as it exists, but we must develop specific methods and perspectives which attest to the fact that it can be changed. If educational research is to become helpful and relevant to the classroom teacher, then we must engage in some demonstration projects. We must test our theories, models, and results in actual classroom settings. We must implement successfully problem solving activities, encourage women's participation in mathematics-related fields in a way which sustains them through the educational system, and supply ways of improving student understanding. Else, we should not continue to advocate these methods in research and in teacher education courses, leaving teachers feeling that failure to implement those methods is their responsibility.. It was the

belief of those of us in SummerMath that in the optimal setting of SummerMath, independent from traditional curricula restrictions, removed from the credit and grade system and with a low student-staff ratio, even with severe time constraints, we could make a difference and we could begin to develop curricular and instructional models which were compatible with our fundamental beliefs about the results of educational research.

Two traditions of research exerted profound impact on the program's design: the research on mathematics learning and the research on women's participation in mathematics. To describe how the research programs influenced the SummerMath program, I will hastily describe what I think we know quite confidently from this work, the tenets we use to interpret and to unify those results which formed the program's philosophy, what we did as a result, and our assessment of the effects.

Research on Mathematics Learning

I will begin by discussing the work on the cognitive processes in mathematics learning. In doing so, I hope to present a list of claims about the research which has been conducted across the various paradigms of cognitive psychology, information processing, and mathematics and science education. From these, I will show how a constructivist interpretation of this research has good explanatory and predictive power and is relatively parsimonious. I think that we can agree that the following descriptions of the state of mathematics education are accurate:

1. According to achievement tests, such as NAEP, students perform significantly poorer when asked to solve non-routine problems. Their application of computational skills to word problems is also weak (Carpenter et al., 1980).

2. Student errors are not random, but are often predictable. This implies that they have a rational basis. Quantitative studies of these errors frequently provide examples of reasoned and not capricious errors (Ginsberg, 1977; Radatz, 1979; Erlwanger, 1975).

3. Specific topics such as rational numbers, proportions, word problems, functions, equality, limits, and variable have fairly extensive documentation that students possess idiosyncratic meanings for the concepts. These may be either alternative conceptions from those standardly presented or misconceptions (Schwarzenberger and Tall, 1980).

4. In interviews, students describe their reliance on memorizing rules and procedures for solving problems. They further indicate their dependence on imitating examples in the text. Researchers have interpreted this to mean that there is

inadequate development of the fundamental concepts (Confrey and Lanier, 1980; Confrey, 1980, 1981).

5. Poor understanding of concepts is also accompanied by weak strategies for approaching mathematical ideas. These weaknesses show up in various descriptions of mathematical processes (Krutetskii, 1976), problem solving skills, and representational systems (Lesh et al., 1980).

6. Students hold conceptions of mathematics and mathematics learning which discourage conceptual development and the strengthening of the processes. They focus on the importance of getting the correct answer; they manipulate blindly without appealing to their intuition, to a representation, or to a concrete case; and they see mathematics as a rigid formal system upon which they have little or no personal claim (Confrey, 1981).

7. Students are unaccustomed to thinking about their own thought processes. When asked to solve problems out loud and reflect on what they have done, they frequently spontaneously respond with pleasure to the novelty of the situation (Lochhead).

In reviewing these statements, the situation for the learning of mathematics seems dismal indeed. This would indeed be so if it weren't for the accompanying pleasures of listening to students solve problems. Researchers in these traditions share the joys of hearing how original, spontaneous, and insightful students can be when provided opportunities to describe their thoughts; and these researchers are becoming aware of how students attempt to make sense of mathematics--even if the results of those efforts are not the ends for which we hope. We hold the firm belief that it is not students' lack of willingness or ability which hampers the development of mathematical insight, but rather the lack of opportunity and encouragement.

Perhaps the last statement is overstated. The ties between these seven descriptions of students' understanding of mathematics and instruction as it takes place in classrooms are unknown. Very little research has been conducted that connects these outcomes of our educational processes to mathematics classrooms. Research on the activity of teaching and learning in classrooms, when it has focused on particular subject matter, has usually accepted achievement test results as its outcome measure (Good, 1980, 1979; Evertson, 1979). In my opinion, from a fusion of these two traditions, new and profound research can result.

But this is not my task at hand. If these seven claims about students' understanding of mathematics are warranted from the evidence, if their selection represents the current research, and if their presentation is in a language which is acceptable across research paradigms, then I can begin my next

undertaking. I want to suggest that alone they do not prescribe action. From them one can conclude that many actions are possible. One can argue that the best action is a model of direct instruction wherein a teacher emphasizes concepts and processes, provides non-routine problems, models the use of representation and concrete examples, and so on. (Good and Grouws, 1979). Or, one can argue for a completely individualized program--better programmed instruction which uses our knowledge of student errors (Bork, Plato). Thus, I want to suggest that in order to translate the research into practice, a philosophy or a theory must be used. It must explain these seven descriptions and predict other occurrences not yet examined. If the theory is quite powerful, it should also be possible to begin to build bridges to other subjects. From this, one can begin to chart a course of action. And action, I must hasten to remind myself, is always a response to the current situation; and it is here that practice again comes into play. The question becomes does your research allow you to act in such a way as to make sense of practice, and does it predict what will happen and guide future action.

I want to suggest that a constructivist approach provides such a philosophy. To describe this philosophy I will describe the most fundamental tenet of it, as stated by Glasersfeld (1974):

A person's representation of the environment, his/her knowledge of the world, is under all circumstances the result of his/her own cognitive activity. (p.22)

This constructivist view of knowledge is in sharp contrast to views that try to reveal some ontological reality rather than to explain how we come to see the world as we do. The implications for teaching and learning are vast; it affects how we portray all four of Schwab's "commonplaces" of schooling: the learner, the subject-matter, the teacher, and the context of schooling.

Within the program, four major tenets followed from this basic assumption:

1. Students, using their previous representations and knowledge, will construct a meaning for what they are learning, and the development of more elaborate and unifying constructions should be encouraged.

2. Our capacity for reflection upon our previous representations and the construction of these representations is essential for developing increasingly adequate constructions and evaluating the success of those constructions. This capacity needs more direct attention in instruction (Confrey, 1981).

3. The current portrayal of the subject matter of

mathematics in classrooms is inadequate to promote the development of effective personal constructions. It is portrayed as rigid, absolute, and external, not as a human construction. For instance, problems are not defined against an objective discipline but must be defined as roadblocks to where a student wants to be (Confrey, 1981).

4. Teaching, according to constructivism, requires the nurturing and promotion of more adequate and effective constructions. Although teachers cannot provide constructions for students, they can select what they hope and evaluate to be likely problems to promote the refinement, unification, or elaboration of students' constructions. Teachers, according to this model, are very active, constantly anticipating, interpreting, and testing out their hypothesis about students' construction by questioning and providing potential problems.

5. An individual will also construct representations of the community of others in which s/he resides. Classrooms and schools are such communities, and within these the sharing of personal constructions is undertaken. One can only see another's construction through one's own. Ethics evolve from the perceived relationship of one's own construction and one's perception of the consensus of the collective constructions of others. For example, one's standards of the adequacy of one's own construction is influenced by one's perception of the collective standards of a group. This is a complex issue for constructivism, but one which needs attention. For our purposes, we were primarily concerned with explaining the impact of what is perceived by a student as the community's standards for appropriate performance in mathematics on a student's personal constructions. Issues surrounding terms like authority, power, responsibility, intellectual honesty, and justification are embedded in this tenet.

Given the previous statement of the philosophical basis for the program, we were left to decide what curriculum and instructional methods would be most likely to encourage constructivist learning. I will summarize these.

Curriculum

In writing program materials, we paid close attention to the selection of topics. Because of the students' overreliance on blind manipulation, we chose to avoid algebraic topics for which we could construct no alternative method of insight. Whenever possible, topics could be approached through alternative modes of representation which included: 1) generalizing number patterns; 2) geometric representations; 3) graphing; or 4) experimentation through manipulatives.

We selected both topics to revisit and to explore for the first time. By doing so, we hoped to convince students that

they knew less than thought and they knew more than they thought. In revisiting, we chose topics like the area of polygons or slopes of lines, and in exploring we introduced intuitive concepts of limits and trigonometry.

Concepts were stressed and algorithms were constructed and practiced. We talked about what we meant by mathematical concepts and contrasted it with how they had approached mathematics previously. They demanded rules; we refused. They pulled fragments of procedures from memory; we pressed them into admitting they didn't understand them. Together we tore down meaningless routines and slowly began to replace them with personal constructions.

In the materials, students were asked to write out their generalizations both in English and in symbols to help them see how terse mathematical symbolism is. In seeing if they understood a concept, both examples and non-examples were integrated.

The materials tried to introduce concepts developmentally. Often a problem was presented in hopes of creating a need for the concept and to act as an advance organizer. Often these problems were chosen to conflict or contradict with widely held beliefs about the concepts, akin to what Posner (1982) describes as "leading one down the garden path." We wanted students to clearly articulate their beliefs and commit to them before we provided what we hoped was to be an inconsistency for them. Other problems were selected for their potential to unify various concepts such as the use of the relationship between pi, the variables circumference and diameter, and the concept of slope.

Krutetskii's processes of generalization, flexibility, curtailment reversibility and information gathering (Confrey and Lanier, 1980) are emphasized in our materials. Word problems were used frequently to increase the accessibility of students' knowledge, to demonstrate the importance of flexibility and comprehension, and to stress the importance of application of knowledge.

The use of LOGO in the computers classes was an important part of the program. The beauty of LOGO is its creation of a slice of the world that students can manipulate and explore and which is rich in potential for mathematical insights. Using LOGO contributed tremendously to the constructivist goals for the program; students set their own goals, learned to think analytically, and obtained immediate feedback on their endeavors

They learned to expect to debug programs, and that encouraged them to be more persistent in their pursuit of mathematical ideas. They became less timid about the machines as they realized that they might lose programs or make mistakes, but that the machines would survive their initial

ignorance. LOGO is an example of a microworld that nurtures a student's sense of empowerment as she selects the problems to explore and sets the standards for success.

Instruction

The staff of SummerMath is committed to the belief that how one learns mathematics exerts a profound influence over what one believes mathematics is. For this reason, the implications of our constructivist perspective had significant impact on the methods of instruction within the program. We wanted not only to teach students mathematical concepts but also to teach them: 1) how to learn mathematics more effectively, and 2) how to be more aware of their own thinking processes.

To do so, we used a variety of innovative instructional methods. The primary goal at the beginning was to move the center of authority in mathematics from us, the instructors (or the textbook), into the student herself. Surrendering one's own responsibility to evaluate the success and worth of what one is learning is incompatible with our values for constructivism. Students are still constructing when they do this, and perhaps adapting to what they believe are the demands of mathematics instruction in a typical school setting, but it's a weak personal construction. Unless they can successfully avoid mathematics, it will result in severe restrictions on their capacity to represent their experiences.

For example, we used the method of paired-problem solving (Lochhead, 1977). In these methods, in order to communicate to students the importance of active learning the students work in pairs solving problems. One student acts as the problem-solver and the other is the questioner. As the problem-solver solves the problem out loud, the questioner must follow the explanation, asking questions until the method, right or wrong, is clear. The assumptions underlying this method are three:

1. Clearly explaining one's method can often result in increased precision.

2. Errors frequently have a rational basis, and until that basis becomes clear, the remedy seldom succeeds.

3. Students need to become more aware of their own thinking processes.

We treated "errors" as opportunities to gain entry into students' personal constructions. We explored the errors with students asking them to solve a variety of problems as we tried to find a construction that explained the students' responses. We learned to think of students' "misconceptions" from an advanced scholar's point of view, not as "wrong" in an

absolute sense, but as "inadequate for the purpose of . . . ."

When students asked a question, we responded with a question. This meant that there might be a considerable wait before we responded as we tried to figure out what the student might be thinking. The role of teacher changed dramatically. We grew increasingly less uncomfortable saying, "I don't know; let me think about it" and frequently requested, "tell me more" as we redefined our roles and became accustomed to original, insightful, and occasionally off-the-wall suggestions by students. We discussed explicitly their beliefs about learning mathematics. We talked about memorization and its results. We tried to communicate that we knew changing survival behavior was a difficult undertaking and that they would feel insecure and inadequate at first. As Dr. Romney, our clinical psychologist, had warned us, "With maladaptive behavior, things often get worse before they get better."

We gave no tests except for placement and evaluation of the program, and no grades. The reason for this is that we were stressing cooperation rather than competition. We wanted students to learn to work in groups so they could learn from each other. The groups and pairs strategies seem to me to be essential to avoid the threat of these methods' becoming overwhelming to the teaching in self-contained classrooms.

Most importantly perhaps of all, in my mind, we minimized the pressure which pace exerts on the traditional classroom. Students were encouraged to pursue a topic until they understood it to their own satisfaction. Cognizant of the research on teaching indicating that a brisk pace is conducive to better achievement scores, we pressed on students to work at a reasonbly high level of involvement. Of course, with an instructor/student ratio of ten to one, we were able to assess students' involvement and to encourage consideration of new but related topics.

Lest it seem that all of these innovative curricular and instructional methods were easily implemented without struggle, hesitancy, difficulty, or soul-searching, let me relate a story about what occurred two weeks into the program. By Thursday of the second week we had a significant portion of the students confused, frustrated, and uncertain. This was not what they expected, and it was not mathematics. The adjustment to this new set of demands had them in turmoil. Our staff met for three hours to decide whether to return to a more traditional program. Most keenly, we debated whether to return to more traditional curricular presentations while insisting on the innovative instructional methods trying to progress to our goals in a more stepwise fashion. We listed their complaints:

1. What is this curriculum good for?
2. This is not real math.

3. This is too easy.
4. This is boring.
5. How will this help me in my: (1) grades, (2) next course, (3) SAT's?
6. I don't know what I am going to learn.
7. This is not what I expected.

When we compared these comments to what we were seeing each day in the classrooms, we conceded that it was not that they were finding what we were asking too easy, but that they didn't know how to construct ideas for themselves. They were evaluating the program according to their previous models for learning mathematics and expecting us to interpret the problems and to tell them the answers. They were not yet seeing the problems as their own, but as ours. Most of all, they were confused as their expectations were not being met.

We called a meeting of the whole group. I led off discussing the research on expectations and the impact of one's expectations on subsequent experience. My staff followed, selecting particular questions and addressing them from the perspective of their course and from the philosophy of the program.

The turnabout was dramatic. From then on we had the students with us--they became enthusiastic and confident. Why it happened, what made the difference, I can't say. In my introductory talk and in the brochure, students had the opportunity to hear the same kinds of statements. Did they simply learn a new sort of jargon? Or was the issue one of readiness and conceptual change? I can't say what accounted for the change in their attitudes and in their engagement in the problems, and I hope through journals and interviews to study this summer.

I interpreted that experience to mean that people who believe in this approach to mathematics learning should not underestimate how radical a change is required by it. Each of Schwab's commonplaces of schooling are affected. The roles of teacher and student are redefined; what is taught and how it is taught are changed; and the purposes and contexts of schooling are transformed.

Women and Mathematics

The second tradition which influenced the design of the program was the examination of the factors influencing the participation of women in mathematics. Relying predominantly on the work of Fennema (1977) and Armstrong (1979), we considered the following results from research:

1. Differences in achievement between young women and men are increased during the secondary school years.

2. Differential course-taking accounts from some of this difference, but not all.

3. Students with positive attitudes toward mathematics are more likely to continue electing mathematics courses.

4. Students who believe that they will need mathematics in their careers or education are more likely to continue.

5. Encouragement from parents, teachers, counselors, and peers can influence a young woman's decision to elect to continue mathematics.

The impact on the program of this research was significant. The first decision we faced again demonstrates the importance of underlying philosophical commitments in interpreting research. After much debate we chose to limit the program to young women. Since these students, for the most part, were going to return to coeducational classrooms, we considered teaching them to engage in mathematics together with young men could be important. Furthermore, we were sympathetic to the assumptions of Fennema et al. (1981) underlying their program "Multiplying Options and Subtracting Bias" wherein they point out, ". . . since female attitudes are influenced by their social environment, the attitudes toward females as learners of mathematics held by significant others would also need to be changed if females' attitudes were to be changed." (p.3)

However, because of the commitment of the College to providing opportunities for special academic pursuits for young women, we chose to remain open to only young women. After seeing the aggressiveness with which these young women chose to learn computing while hearing how other schools continue to have microcomputers dominated by males, I think that we made the correct decision. By providing an atmosphere which encourages and supports the pursuit of mathematics by young women, I believe our students return to their classrooms determined and committed to continued participation in mathematics.

We chose to interpret the findings of the research to suggest or be compatible with the following strategies:

1. We provided strong role models of women in mathematics. Besides having a female director, four out of six staff members were women. Five undergraduate mathematics majors also aided in the instruction.

2. In the Mathematics Workshops, and in special evening lectures, we demonstrated the usefulness of mathematics in a variety of careers, including architecture, astronomy,

geology, educational psychology, underwater acoustics, computers, and planning. At a special workshop on women and mathematics, we gave an introduction on the research findings on female participation, and five women in mathematics-related fields described their passage into their careers.

3. Special seminars on mathematics anxiety were offered once per week by Dr. Romney. She used many of the ideas in the work summarized by publications of the Center for the Study of Anxiety.

4. At the end of the program, students were given a summary of Lucy Sells' data on mathematics as a "critical filter" from the University of California at Berkeley. It was emphasized that decisions on courses often become implicit decisions on careers.

All of these strategies were well received by students. What is missing from this description is a translation of the implications of the recent research on differential patterns of attribution by young women and young men and on their differential treatment in classrooms (Wolleat et al., 1980). We hope to see this research develop into providing more successful strategies for intervention. We are also intrigued by the potential merging of these lines of research with constructivist thought. This is described in more detail in the section on "Future Directions."

Evaluation

To evaluate the success of the program a variety of measures was used. We administered pre- and post-tests of achievement in arithmetic and algebra. No significant gains were found in either measure. Two measures of spatial ability showed improvement; however, the effects may be primarily due to practice. We also administered a sample of items from Armstrong's study of women and mathematics. We saw significant improvement in the participants' confidence in mathematics, and our students were significantly less math-anxious by the end of the program. No differences were found in their enjoyment of mathematics, their attitudes toward women who were successful in mathematics, or in their perception of the usefulness of mathematics. The measures indicated a slight increase in the students' perception that mathematics is a male domain and in their tendency to stereotype according to sex. However, both of these measures had low means to begin with, 1.57 and 1.72, on a scale of 1 to 5. A more detailed analysis of this data is under way by Anne Ducharme, a Mount Holyoke student.

Material for evaluating the SummerMath program was collected via two student questionnaires, one completed

immediately following the program and another sent to the students six weeks after they had returned to school.

The first part of the evaluation asked students about their opinions of the approach used in learning mathematics in the program. Specifically, we asked their opinion on the revisiting of curricular topics, the methods of paired problem solving, the adjustments they foresaw necessary on returning to school, and whether they were more aware of the mathematics requirements in various careers.

When we asked the students if our attempts to get them to see behind the scenes on particular topics were successful, sixty-seven percent of the students indicated that they saw significant, positive differences. Thirteen percent more of them wrote that they saw positive differences, while thirteen percent were ambivalent, and seven percent saw differences which they did not prefer.

When asked to specify topics which they found most insightful, the students showed a preference for our treatment of slopes, the Pythagorean theorem, the introduction of pi, and trigonometry. Also mentioned were the word problems and the emphasis on geometry.

The students who felt positive about the approaches focused their comments on the sense of power and freedom they felt by not having to rely so heavily on memorization. They indicated a willingness to persevere more on a problem, an ability to see a unity among the ideas, a satisfaction from having reasons to believe in mathematical truths. They wrote, "I think above all I noticed relation and unity between mathematical concepts" and "The difference was the way we learned it; it was memorized whether it was right or wrong. But the way you presented it, for us to convince you of the fact that it was true, made a big difference." Sadly, one student wrote, "Our teachers don't care about us understanding math concepts; they are more concerned about us passing their exams."

Of the students who did not prefer our approach, most indicated that the pace seemed too slow or that they already knew the information behind the scenes from their previous instruction. One student wrote, "To remember how to do the procedures is harder than to remember the formula." For this student, we clearly were not successful.

We asked students if they foresaw changes they would have to make on returning to study mathematics in their schools. Fifty-seven percent said they would make major changes; twenty-five percent indicated the need for minor adjustments, and eighteen percent said they foresaw no changes necessary.

Of the majority who expected significant adjustments, the students predicted a need to ask more questions in class, to

look deeper and longer into problems, to visualize the problems, to look at the concepts and not just the formulae, and to demand reasons. A number of the students commented on the need to work at one's own pace and the difficulty of doing so in their regular classrooms. One wrote, "We have to find someone patient enough to ask and discuss the reasons why . . . so we can understand it better. Sometimes it takes a long time."

Examples of the comments by students who expected to have to make changes in their behavior follow. One student commented, "I will have to learn to continue to be aggressive with my ideas and questions, and not to be satisfied with a correct answer, but with an understanding I can call back to." Another student wrote, "When a problem is presented, do not go into a panic. Ask yourself questions, experiment, and when you find an answer, ask yourself how you got it." Summarizing the fundamental tenet of all we tried to teach, one student wrote, "I will definitely take more time analyzing the concept rather than plugging in the numbers."

Some students indicated concern that there would be a conflict with the methods of instruction in the regular classroom. Some indicated a determination to change their own behavior, such as one student who wrote, "I will convert what the teacher is saying into a more logical way of thinking. It will mean more work because now I will almost always have to listen to what the teacher says, then reteach it to myself." Others are less confident: "I think that going back to school will be very frustrating. Now that I have seen a little bit of what mathematics can be, I never want to go back to memorizing formulae and solving everything the teacher's way. I will have to keep remembering what I learned here and not let myself slide into old habits. I am also going to demand more from my math teachers, especially in explaining problems." Some students indicate that they will respond to the conflict by accepting the existing methods. One such student wrote, "Even though I learned a lot here, when I go back I will have to memorize and accept things without explanation, at least the ones that are new to me."

Of the students who indicated no need for change, the responses varied. One wrote, "I may be thinking differently than before, but the end result . . . will be the same." Another said confidently, "All the changes have been made already." Or one wrote, "I feel that through your instruction I am able to think more analytically and organize my thoughts better. As for changes, I won't make any; I will employ what I learned here."

When asked for their opinions about the methods of paired problem solving and small group instruction, fifty-six percent of the students said they liked both, twenty-eight percent liked only one type, and sixteen percent disliked the methods. More students were enthusiastic about working in pairs than in

small groups. The responses given to this question seem to reflect the importance of considering the sociological aspects of classroom learning as well as the cognitive implications.

The students who liked the paired problem solving wrote comments such as the following: "Working in pairs forces one to explain to her partner, which helps in one's own thinking process. Also, listening to a partner gives one a better understanding of her partner's thinking process. Some of the advantages of paired problem solving are the thought you must put into everything you do, so that you can justify your manipulations to your partner. Being the listener teaches you to question every move and think in logical steps. I think both people become more active in the learning process." Many students liked the method if they were paired with a student who learned at the same pace they did and who shared the same knowledge base.

The students who were less enthusiastic about the instructional methods often raised concerns with the small group work because of the homogenous grouping and further gave some indications that there were peer pressures not to be too studious or too purposeful. Two students expressed a preference in working alone; as one wrote, "I do not work well with other people at all, and this approach was very hard for me, even though I need to learn to work with other people." Fears about revealing inadequacy or ignorance were discussed repeatedly. However, in many of these cases, the students still supported the method and occasionally made comments such as, "It's easier to work with someone your own age, because they're more on your own level."

The participants were asked if they had become more aware of the applicability of mathematics to various careers as a result of the program. Sixty-one percent of the students indicated that they were more aware, and thus, thirty-nine percent indicated that they were not more aware. Characteristic of the comments by those students with increased awareness was the following statement: "I realized that math relates to almost everything in the world. I find it fascinating to look around you and see math everywhere. I especially like this part of the program, and I now realize that I want a career in some type of applied math like engineering." Many of the students expressed an interest in computing as a result of the program. Since many of the career exploration activities were voluntary, some students didn't gain all the benefits from this part of the program. A particularly successful evening program was the one on women and mathematics, and various students mentioned this in their comments.

Students who had already decided on a career often said their horizons were not broadened, which is a curious response. One such student wrote, "No, but I came here wanting to be a doctor, and I still want to be a doctor." One

student had her heart set on a career which she felt was inaccessible to her because of her weak mathematics. She wrote, "Before I came to SummerMath I was considering going into some other field rather than veterinary medicine, as I have always wanted to do. I was very discouraged about my math in high school and scared to go on in college. Now I have more confidence and I am going into medicine as I wanted to."

We asked the students for their suggestions for improving the program. The suggestions which were made repeatedly were to group the classes on previous knowledge, to offer shorter classes, to explain earlier how our goals differ from traditional mathematics classrooms, and to reorganize the Mathematics Laboratory. All of these suggestions, except perhaps the length of the classes, will be enacted in various forms, and the quality of these suggestions reflects the seriousness with which these students undertook the evaluations.

The final question in this section asked the participants if they think of mathematics in the same way as they did six weeks earlier. Sixty-eight percent of the students said that there was a positive change in their way of looking at mathematics. Eighteen percent said there was no change, and five percent said they felt more negative about mathematics. The positive responses indicated that the students now find mathematics more challenging, more creative, and that it can be fun. Others said they were more confident and flexible with mathematics and more likely to persevere in solving a problem. Others repeated their understanding that mathematics is more than just formulae. Three comments represent the range of such responses: "Six weeks ago I didn't want to get on the plane to come here. Now I don't want to go home. For the first week I was scared to get up and go to class because I could never get the right answer in school and I was afraid someone would call on me. I always thought of math as being very mechanical, devoid of curiosity and creativity. Now I know better." "Mathematics can be for thinking and exploration, but that takes time. I've learned to take my time, because if I try hard enough I can solve any problem." "I am more aware now that mathematics is not such an unattainable subject--but something that we can learn and deal with every day of our lives."

Six weeks after the program ended, participants in SummerMath 1982 were sent another evaluation form to complete in order to assess longer term effects of the program. Thirty-six students returned the questionnaire, which included a return envelope addressed to the secretary to preserve their anonymity and a section for their parent(s) to complete. A summary of the students's responses is presented in the next section.

One hundred percent of the students responding indicated that they were enrolled in a mathematics course. Students were asked to rate their level of anxiety prior to entering the program and again after the program had ended to see if they felt less anxious about mathematics. The rating scale went from 1 (most anxious) to 5 (least anxious). The mean score prior to attending the program was 3.6 (standard deviation was 1.3), and it was raised by .5 points to 4.1 (the standard deviation was 1.0) after the program according to the students' assessment.

According to the students' assessment, improvement in students' perception of their potential to do well in mathematics also resulted from the program. Using a five point scale, students were asked to rate their potential to do mathematics before and after the program. The pre-program mean was 3.3 (with a standard deviation of 1.2), and the post-program assessment had a mean of 4 (with a standard deviation of .75).

Students were asked to respond to the question, "In your current mathematics course, your experience in SummerMath has:

1. You are not taking mathematics.
2. Had no effect at all or has been detrimental to how you learn mathematics.
3. Had little effect on how you learn mathematics.
4. Had a significant positive effect on how you learn mathematics.
5. Improved dramatically how you learn mathematics."

Eighty-three percent of our graduates responded to this question with a rating of 4 or better. The mean response was 4.0 with a standard deviation of .57.

We followed this question by asking students to elaborate on their responses if they chose to do so. The following are excerpts from those responses:

The SummerMath program has helped me understand better derivatives in calculus.

In saying that my experience at SummerMath has been detrimental to how I learn mathematics, I don't mean that I now feel less confident. I mean that now that I've seen how well math can be taught I'm frustrated with the way my teacher teaches it. He doesn't teach badly, but I realize that he could teach it better than he does. I am able to do the work and get good grades, but I don't really feel that I know what I'm doing when I write out formulas, identities, etc. I can't see it in my mind. Going to SummerMath has helped me a lot, though, as well. I had only had Algebra II before this year, and I was planning to, and now am, taking trigonometry and geometry. The geometry and trig that we did this summer has

helped me in the trig for which I have not yet had the geometry.

Every once and awhile the teacher asks a tough question, expecting no one to respond, and I can answer it.

My current math course is programmed. There are no teachers. Our only resources are some sorry graduate students, some of which speak very little English. The class has a 50% drop rate, and only 25% of the students pass. Before I went to SummerMath I would not have been able to complete a course like this. Currently I am receiving a high B or A.

The change I have noticed in how I learn mathematics is in approaching a problem from more than one direction. If I can't solve a certain problem by following examples of similar ones, I will now go back and try it from a totally new direction. Starting from scratch I use my knowledge of very basic mathematics to work up to the level of the problem and then, hopefully, solve it. Sometimes I succeed, sometimes I don't. Either way I usually end up learning something more about the type of problems and ways to solve them that we are currently studying.

The SummerMath program gave me confidence and a desire to learn mathematics. I learned how to conceptualize this summer, and that has been the most beneficial aspect.

I look at problems, especially word problems given, more analytically.

SummerMath has not helped me in math, but I have found that it is helping me in chemistry, concerning logically solving problems.

When the teacher does the trig proofs on the board, I don't sleep; I look at what he's doing and see how it relates to the product theorem or formula.

It has given me more patience and incentive to figure out problems.

When I do not understand something the first time I see it, I'm not as willing to instantly give up as I used to be. Usually if I look at it for five more minutes it comes to me.

I am eager to understand it more and not satisfied with just memorizing formulas and steps to take.

Well, I'm not afraid of math as I used to be. In most cases I feel I'm able to do it, and that does help me understand it better, because I want to; but I have to go for help every day. However, I'm having a great deal of difficulty in physics with the word problems.

Before SummerMath I enjoyed math--but I did not strive to do well. Now I love math and I want to do well in it. It is enjoyable for me. As of right now I am not quite sure how SummerMath affected my learning ability in math.

One of the goals of the SummerMath program was to change students' conceptions of mathematics. Many students come to SummerMath believing that getting the correct answer is far more important than understanding the processes involved in obtaining that answer. In doing problems, they would insist on one method and be satisfied with memorizing a single formula for that method. We tried to teach students that mathematics is for exploring and discovering, and that the processes involved in generalizing, guessing, testing, and concluding were a legitimate part of the mathematics.

To see if we were successful, we asked students to rate a series of statements from 1--strongly disagree to 5--strongly agree. (In describing these results, all the questions are adjusted so that a high score indicates a conception of mathematics more in line with what we hoped to achieve. However, they were asked in various directions.) The question and the means and standard deviations of the responses are given below:

1. Mathematics is important for my future. The mean was 4.7 (with a standard deviation of .71).

2. How you get an answer is as important as whether the answer is right or wrong. The mean was 4.4 (with a standard deviation of 1.0).

3. There is more than one way to get a right answer in mathematics. The mean was 4.2 (with a standard deviation of 1.3).

4. Computers are not smarter than people. The mean was 4.1 (with a standard deviation of 1.0).

5. Mathematics is not a bag of tricks. The mean was 4.0 (with a standard deviation of 1.0).

6. Mathematics is not done best all by one's self. The mean was 3.7 (with a standard deviation of 1.1).

We asked students about the effect of the SummerMath program on their career awareness. Fifty-six percent said that they learned more about the mathematics required for many careers. Thirty-six percent said that they already recognized the importance of mathematics, and eight percent said that they did not learn any more about careers and mathematics.

Future Directions

The interactions between the past research, as it exists in articles, books, and through the spoken word, and the present practice of SummerMath are complex and enlightening. Research, so far as it is helpful in this endeavor, must help us to make sense of what we are seeing in the rapid pace of the program. It must be interpreted along with its implicit assumptions and values to suggest practices which complement it. Research surely exerted its influence on the progam, and its impact was positive and unifying.

Practice, however, was not simply the passive recipient of this force; and certain problems recurred in persistent patterns--problems for which our current guide or map from research provided no explanation or language by which to describe those events--and so, subtly, stubbornly, these areas tugged at our awareness. It is only now, nine months after the clamor of the program has subsided, that I offer new guesses of some of the phenomena which were so perplexing. Undoubtedly much more was also missed, experience so foreign to our traditions of looking that we cannot even speak of what it is.

In this last section, I want to present a set of new issues to be addressed by researchers in mathematics education. Some came from taking existing research and placing new demands on it, such as asking of the research on misconceptions, what is called for in terms of teaching? Others have evolved from juxtaposing the two traditions, research on women and mathematics and research on misconceptions.

1. Constructivism provides a way of describing how one comes to view the world as one does, but it does not explain adequately why one learns. Self-motivation or curiosity is quite difficult to explain, and larger social issues such as for what purpose should students study mathematics have not been adequately addressed. In particular, questions about poorer performance in mathematics by females than males and by minorities than Anglos must be investigated. In summary, through constructivism we must begin to examine some of the questions presented in the new society of knowledge on the distribution of educational opportunities in mathematics. Our context for examining these questions needs to broaden beyond the level of the individual student.

2. A more specific application of this question involves the integration of the work on attribution theory and constructivist approaches. I want to see us examine the relationship between our approaches to teaching mathematics and math-anxiety. The women and mathematics research must enter into the classroom processes and challenge the implicit assumptions held by teachers, parents, and students about mathematics ability.

In explaining to parents our view of mathematics anxiety,

I have often said that anxiety results when one is placed into a situation in which one has no control. Powerlessness breeds anxiety, and students who memorize routines and rules and whose knowledge is fragmented and isolated from the problems it solves lack empowerment. In the teaching of learning of mathematics, this is how we treat anxiety.

I left unsaid the nature of the connection between power and knowledge, and an article by Georgia Sassen (1980) provided a possible key. Sassen described the work of Kegan, who wrote, "Our world design or theory or 'structure of knowing' is what allows us to 'construct a meaning' for our experiences and thus to 'have' those experiences." (p.17)

Sassen explained how this constructivist approach can be tied into a definition of anxiety. She wrote:

From this perspective, anxiety takes on new dimensions. Because it arises from the inability to make meaning, and meaning-making is so connected with personal and existential commitment, there is much more to the anxiety experience than fear of what comes next. (p. 18)

This is the union of the two research traditions which provides the promise of hearty synthesis.

3. We desperately need to develop alternative methods of evaluating what students understand and what characteristics of independent learners they have developed. We need questions which require students to approach them from a variety of directions, which promote strategies of debugging, plugging in numbers using representations, and tinkering. These questions must be on content which mathematicians consider essential and mainstream, and the measures ideally will illustrate dramatic differences between blind skill manipulators and problem-solvers.

In addition, we need measures of personal competence and independence in doing mathematics problems. Our graduates spoke of increased persistence, autonomy, flexibility, tolerance for ambiguity, open-mindedness, level of engagement, and critical assessment of their own level of understanding. These measures need to be mathematics-specific if we are to convince our most potent skeptics. We must move beyond convincing ourselves--we know the state of mathematics education, and we have some tentative answers. We need help on developing the forms of proof.

Conclusion

In my reviews for my proposal for this paper, one reviewer wrote that it sounded more like a program description than research. I hope in most people's minds that I have proven him wrong. It is such a myopic vision of research as this which leaves us in danger of becoming irrelevant to the

practicing teacher or struggling student.

In this paper, I have described one case where research has exerted a significant positive effect on a program's design. In turn, the practice of that program has influenced profoundly our understanding of that research. I have described those interactions and discussed how they are mediated through philosophical perspectives. Finally, I have suggested new directions for research based on the interactions of two powerful research traditions and of those traditions coming into contact with practice.

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